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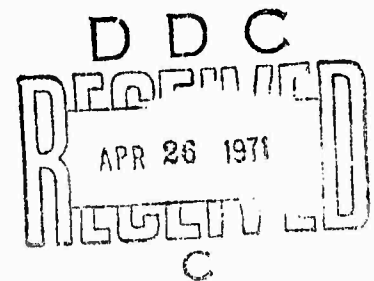
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# A MATHEMATICAL MODEL FOR PREDICTING THE DAMPING TIME OF A MERCURY DAMPER

by

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**ABSTRACT.** The mercury damper is a device for damping the nutation of a rotating body. This report explains how a mercury damper works, and derives a simple expression for the damping time.



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## NOMENCLATURE

A	Wetted area
$C_f$	Resistance coefficient as a function of the Reynolds number
F	Frictional drag force
$H_p$	Component of $\bar{H}$ along $\bar{P}$
$H_t$	Component of $\bar{H}$ along $\bar{T}$
$\bar{H}$	Total angular momentum
$I_p$	Polar moment of inertia
$I_t$	Transverse moment of inertia
L	Length of channel filled with mercury
$\bar{N}$	$\bar{T} \times \bar{P}$
O	Center of gravity
$\overline{OP}$	Symmetry axis
$\bar{P}$	Unit vector along $\overline{OP}$
$R = vLv^{-1}$	Reynolds number
$\bar{R}$	Position vector
$R_n$	Component of $\bar{R}$ along $\bar{N}$
$R_p$	Component of $\bar{R}$ along $\bar{P}$
$R_t$	Component of $\bar{R}$ along $\bar{T}$
$\bar{T}$	Unit vector in plane defined by $\overline{OP}$ and $\bar{H}$ , and directed so that $\bar{T} \cdot \bar{H} > 0$ and $\bar{T} \cdot \bar{P} = 0$
a	Acceleration in direction of F due to motion of coordinate system

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$m$	Mass of mercury
$r$	Lever arm
$t$	Damping time
$v$	Peak velocity
$\gamma$	Angle between $\bar{H}$ and $\bar{P}$
$\gamma_f$	Final value of $\gamma$
$\gamma_i$	Initial value of $\gamma$
$\nu$	Kinematic viscosity
$\rho$	Specific gravity
$\bar{\omega}$	Total angular velocity
$\omega_p$	Component of $\bar{\omega}$ along $\bar{P}$
$\omega_{pi}$	Initial value of $\omega_p$
$\omega_t$	Component of $\omega$ along $\bar{T}$

## INTRODUCTION

A mercury damper is a device consisting of a circular channel partially filled with mercury fixed to a rotating rigid body so that if any nutation of the rigid body occurs, it is automatically removed by the damper. The mercury damper accomplishes this by extracting kinetic energy from the system. The time required to stop the nutation is the damping time, which is characteristic of the particular combination of damper and rigid body.

The following is a simplified analysis of the motion of the rigid body, and of the effect of the damper upon that motion. The resulting equation for the damping time will aid in the design of a mercury damper which will suit the requirements of the designer.

## DESCRIPTION OF MOTION

Consider a rigid body with axial symmetry with the greatest moment of inertia about the symmetry axis, spinning, without the influence of external forces (see Fig. 1). Suppose the body moves in the most general way so that the symmetry axis,  $\overline{OP}$ , the total angular momentum,  $\overline{H}$ , and the total angular velocity,  $\overline{\omega}$ , are non-coplanar.

The location of a point in the rigid body shall be specified by a position vector,  $\overline{R}$ , drawn from the center of gravity,  $O$ , as an inertial reference. An orthogonal triad is defined by the unit vectors:  $\overline{P}$  along  $\overline{OP}$ ,  $\overline{T}$  in the plane defined by  $\overline{OP}$  and  $\overline{H}$ , and directed so that  $\overline{T} \cdot \overline{H} > 0$  and  $\overline{T} \cdot \overline{P} = 0$ , and  $\overline{N}$  defined by  $\overline{T} \times \overline{P}$ .

$$\overline{R} = R_p \overline{P} + R_n \overline{N} + R_t \overline{T}$$

and

$$\overline{\omega} = \omega_p \overline{P} + \dot{\gamma} \overline{N} + \omega_t \overline{T}$$

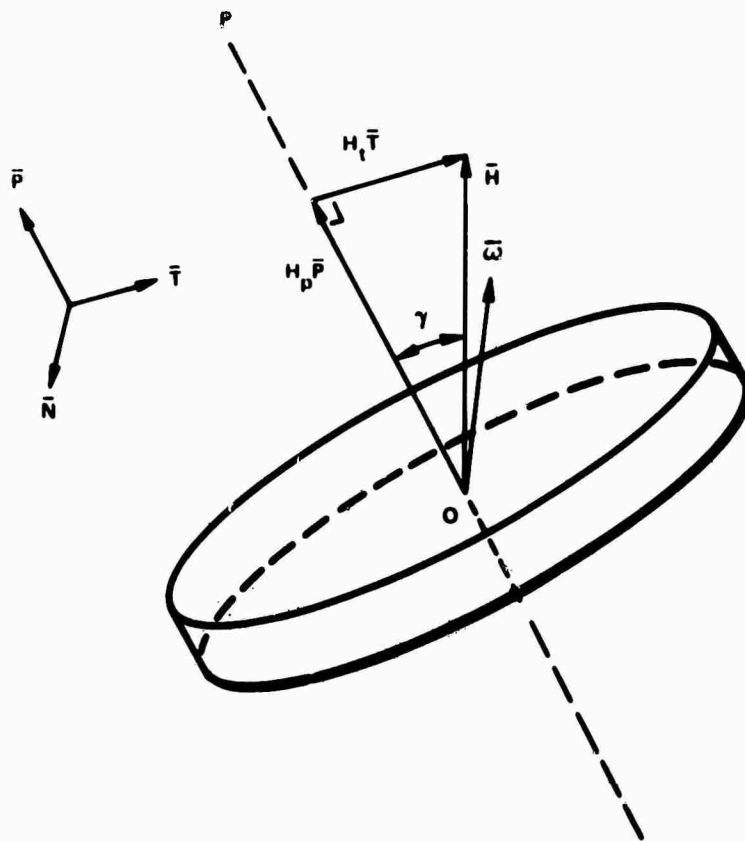


FIG. 1. Spatial Relationships of the Defined Quantities.

but

$$\bar{H} = H_p \bar{P} + H_t \bar{T} \text{ since } H_n = 0$$

$$H_p = \omega_p I_p = H \cos \gamma$$

Differentiating,

$$\dot{\omega}_p I_p = -H \sin \gamma \dot{\gamma}$$

$$\dot{\gamma} = - \frac{\dot{\omega}_p I_p}{H \sin \gamma} \quad (1)$$

The eventual integration of Eq. 1 will yield the damping time for the system.

## EFFECTS OF DAMPER UPON RIGID BODY MOTION

In the problem that is being considered, a coherent mass of mercury is allowed to flow through a circular channel that is coaxial with the rigid body (see Fig. 2). The frictional drag,  $F$ , between the walls of the channel and the mercury produces a torque about  $OP$  which, by Newton's second law, must equal  $\dot{H}_p$ :

$$r \sqrt{R_n^2 + R_t^2} = \dot{\omega}_p I_p$$

Letting  $r$  represent the lever arm, and substituting in Eq. 1,

$$\dot{\gamma} = - \frac{rF}{H \sin \gamma} \quad (2)$$

Note that if the damper were not present, then  $\dot{\gamma}$  would equal zero, and the assumption of the first section that  $OP$ ,  $H$ , and  $\omega$  are non-coplanar would not be true.

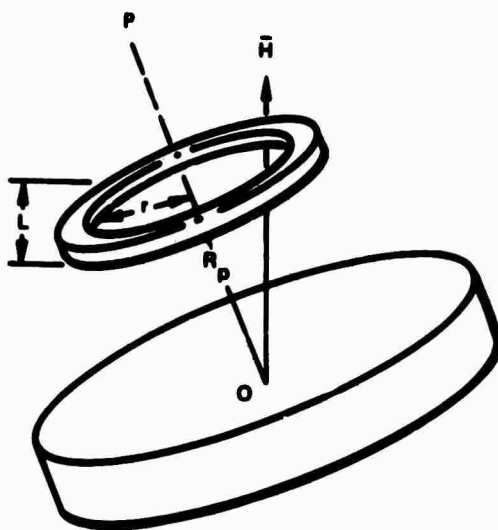


FIG. 2. Damper as Fixed to Rigid Body.

(Note the position of the mercury in the channel relative to the total angular momentum vector,  $\vec{H}$ .)



For the mercury flow rates to be considered, the Reynolds number, given by  $R = vLv^{-1}$  (where  $v$  is the peak velocity,  $L$  is the length of channel filled with mercury, and  $\nu$  is the kinematic viscosity) is sufficiently large so that the flow shall be considered turbulent. Under these conditions, the frictional drag can be represented approximately by the empirical relation:<sup>1</sup>

$$F = \frac{1}{2} C_f A \rho v^2 \quad (3)$$

where

$A$  = Wetted area

$\rho$  = Specific gravity

$v$  = Peak velocity

$C_f$  = Resistance coefficient which is a function of the Reynolds number.

For this problem,  $C_f$  lies in the range  $0.004 > C_f > 0.002$ . The  $v$  is a complicated function of the dimensions of the channel and of the position within the channel. For this reason,  $v$  is assumed to be equal to the average speed of the mercury in the channel, which is approximately true. Now that an expression for the frictional drag is available, the damping time can be calculated.

#### DERIVATION OF THE DAMPING TIME

It is supposed in the analysis that the nutation is violent enough so that the mercury is forced centrifugally to a position in the channel which is far from  $\bar{H}$  (see Fig. 2), as  $\bar{H}$  turns about  $\bar{OP}$  (or  $\bar{OP}$  about  $\bar{H}$ ) at the rate:

$$\frac{\omega_H}{H_t} = \frac{I_p}{I_t} \omega_p$$

---

<sup>1</sup> H. Rouse and J. Howe, "Basic Mechanics of Fluids," p. 170, New York, Wiley, 1953.

However, the rigid body turns about  $\overline{OP}$  at  $\omega_p$ . Therefore, the speed of the mercury relative to the channel is

$$v = r(I_p/I_t - 1)\omega_p = r(I_t^{-1} - I_p^{-1}) H \cos \gamma$$

Combining Eq. 2 and 3, and substituting for  $v^2$ :

$$\dot{\gamma} = - \frac{\frac{1}{2} r^3 C_f A \rho (I_t^{-1} - I_p^{-1})^2 H^2 \cos^2 \gamma}{H \sin \gamma}$$

rearranging,

$$\frac{\dot{\gamma} \sin \gamma}{\cos^2 \gamma} = - \frac{1}{2} r^3 C_f A \rho H (I_t^{-1} - I_p^{-1})^2$$

Integrating from  $\gamma_i$  to  $\gamma_f$ , and replacing  $H$  by  $\omega_{p_i} I_p / \cos \gamma_i$  yields:

$$t = \frac{1 - \frac{\cos \gamma_i}{\cos \gamma_f}}{\frac{1}{2} C_f A \rho r^3 \omega_{p_i} I_p (I_t^{-1} - I_p^{-1})^2} \quad (4)$$

This is the time required for the frictional drag of the mercury damper to cause the rigid body to precess so that  $\gamma$  is changed from  $\gamma_i$  to  $\gamma_f$ . The most useful case is the one for which  $\gamma_f = 0$ , since  $t$  is then the damping time. However, the relation is not strictly true when  $\gamma_f$  is arbitrarily small, since it was assumed that the nutation was violent enough to constrain the mercury to a fixed position in the coordinate system; i.e., to a point bearing a fixed relation to  $H$  and  $\overline{OP}$ .<sup>2</sup> Under these circumstances, the condition that  $F = ma$  must be satisfied, where

$$F = \frac{1}{2} C_f A \rho v^2$$

<sup>2</sup> This is experimentally observed.

is the frictional drag,  $m$  is the mass of the mercury, and  $a$  is the acceleration in the direction of  $F$  due to the motion of the coordinate system. When  $a$  becomes everywhere smaller than  $F/m$ , then Eq. 4 is no longer valid.

The vector acceleration at any point  $R$  is:

$$\ddot{\bar{R}} = R_p \ddot{\bar{P}} + R_t \ddot{\bar{T}} + R_n \ddot{\bar{N}}$$

From Fig. 1, neglecting  $\dot{\gamma}$ ,

$$\dot{\bar{P}} = \omega_t \bar{N}$$

$$\dot{\bar{T}} = -(I_p/I_t) \omega_p \bar{N}$$

$$\dot{\bar{N}} = (I_p/I_t) \omega_p \bar{T}$$

Since  $\dot{\gamma} = 0$

$$\dot{\omega}_p = 0 \quad \text{and} \quad \dot{\omega}_t = 0 \quad \text{as indicated by Eq. 1}$$

then differentiating,

$$\ddot{\bar{P}} = \omega_t \dot{\bar{N}} = (I_p/I_t) \omega_p \omega_t \bar{T}$$

$$\ddot{\bar{T}} = -(I_p/I_t) \omega_p \dot{\bar{N}} = -(I_p/I_t)^2 \omega_p^2 \bar{T}$$

$$\ddot{\bar{N}} = (I_p/I_t) \omega_p \dot{\bar{T}} = -(I_p/I_t)^2 \omega_p^2 \bar{N}$$

Substitution in  $\ddot{\bar{R}}$  yields

$$\ddot{\bar{R}} = R_p (I_p/I_t) \omega_p \omega_t \bar{T} - R_t (I_p/I_t)^2 \omega_p^2 \bar{T} - R_n (I_p/I_t)^2 \omega_p^2 \bar{N}$$

$a$  is the component of  $\ddot{\bar{R}}$  along  $\bar{r} \times \bar{R}$ , where  $\bar{r} = R_n \bar{N} + R_t \bar{T}$ :

$$a = \frac{(\bar{r} \times \bar{R}) \cdot \ddot{(\bar{R})}}{|\bar{r} \times \bar{R}|} = \frac{1}{r} (R_t \bar{N} - R_n \bar{T}) \cdot \ddot{(\bar{R})}$$

Multiplying,

$$a = -\frac{1}{r} R_n R_p (I_p / I_t) \omega_p \omega_t$$

$$I_p \omega_p = H \cos \gamma$$

$$I_t \omega_t = H \sin \gamma$$

$$a = -\frac{R_n R_p H^2}{r I_t^2} \cos \gamma \sin \gamma$$

For a given value of  $\gamma$ , the largest value of  $a$  is that for which  $R_n = -r$ .

Substituting for  $R_n$ , and setting  $a = F/m$  will determine the minimum value of  $\gamma$  for which the observed damping mechanism is operative:

$$F/m = R_p H^2 / I_t^2 \cos \gamma \sin \gamma$$

but

$$F = \frac{1}{2} C_f A \rho v^2$$

Substituting for  $v^2$ , and solving for  $\gamma$ :

$$\gamma_{\min} = \arctan \frac{C_f A \rho r^2 (1 - I_t / I_p)^2}{2m R_p} \quad (5)$$

In most problems, it is found that  $\gamma_{\min}$  is small enough so that  $\gamma_f$  of Eq. 4 can be set equal to zero without significant error:

$$\text{damping time} = \frac{1 - \cos \gamma_1}{\frac{1}{2} C_f A \rho r^3 \omega_{p1} I_p (I_t^{-1} - I_p^{-1})^2} \quad (6)$$

provided  $\gamma_{\min}$  is small

Figures 3 and 4 are graphs of  $R$  and  $C_f$  for use with Eq. 5 and 6.

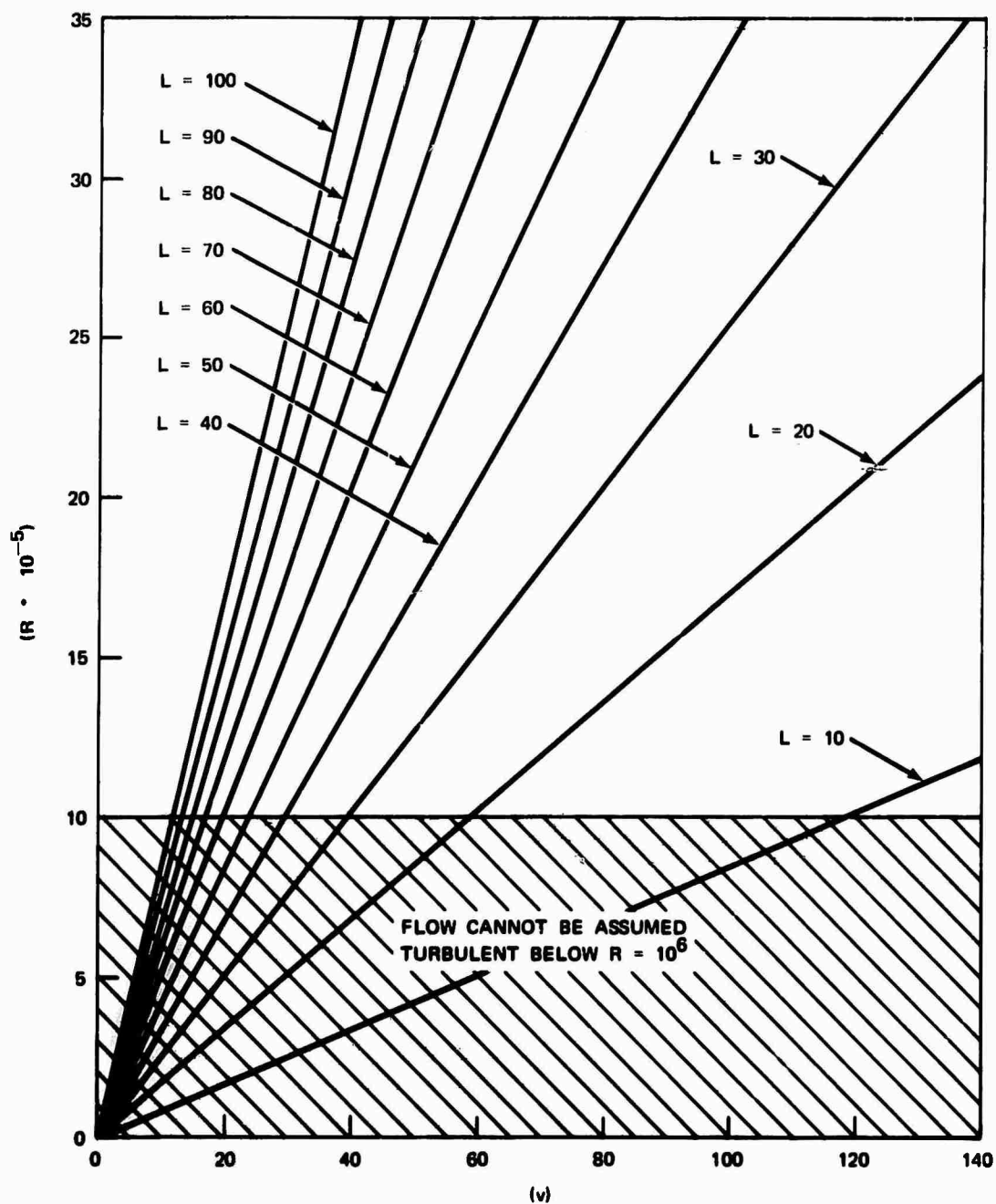


FIG. 3. Reynolds Number,  $R = vLv^{-1}$  ( $v_{Hg}^{20*} = 0.00118 \text{ cm}^2/\text{sec}$ ).

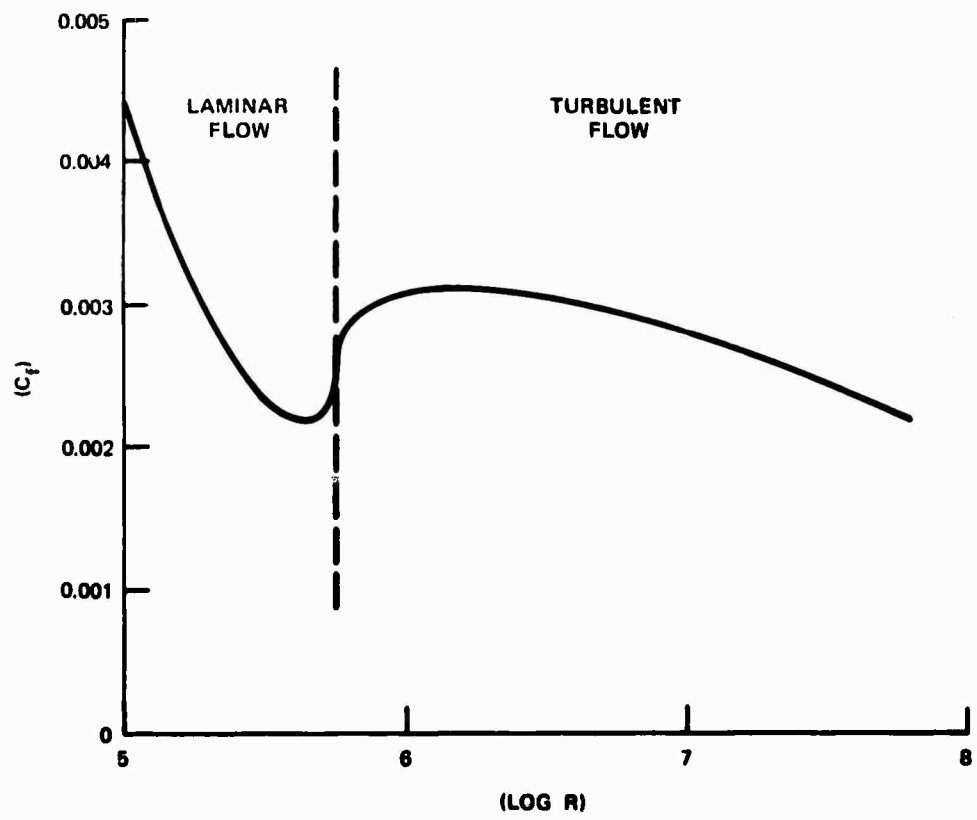


FIG. 4. The Resistance Coefficient,  $C_f(R)$ .

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